HEAT-SHIELDS WITH FACING CURTAINS

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Composite coatings consisting of a refractory facing and an insulating, decomposing backing are considered. Thermal calculations are performed for a plane coating with heat flux constant over the surface and in time.

The theory of the composite heat-shield – refractory facing and insulating decomposing backing – is that the heat from outside passes through the refractory facing and decomposes the insulating backing, the decomposition products then forming a layer with high thermal resistance. If the parameters of the system are chosen correctly, the highest temperature that the facing may reach under given conditions occurs at its inside surface. The temperature drop, and thus the heat flux, have minimum values.

The part of the heat flux passing through the facing is carried away by the decomposition products before it reaches the backing; the remainder goes into heating and decomposing the backing. Decomposition proceeds comparatively slowly, since the heat flux is reduced by the facing and by the flow of decomposition products. Because of its low thermal diffusivity, the backing is heated only to a small depth, the wall thus being safely insulated.

In cases where there is no appreciable penalty in thickness or in weight, a coating with a facing curtain is preferable for parts where the wall contour must remain unchanged.

The thickness of the coating is determined by the rate of decomposition. We shall estimate this and compare the composite coating with the single-layer case. It is convenient to take the case of a plate immersed in a uniformly accelerated gas stream in steady laminar flow. The heat transfer coefficient is then constant along the coating (in the x direction), and the heat flux, gas temperature in the gap, and rate of decomposition of the backing may be considered independent of x. Similar conditions exist in flow over blunt bodies, and in internal flows with mass transfer.

It is further assumed that the process is steady (if the gap is kept constant), or quasi-steady (if the gap varies). Variation of the thermal conductivity λ and heat capacity c with temperature t are not taken into account.

Let us examine the flow of gas in the gap from the thermal point of view. With the given conditions the energy equation has the form

$$c_p v\rho \frac{dt}{dy} + \lambda \frac{d^2t}{dy^2} = 0. \tag{1}$$

On the surface of the backing, the vertical component of mass velocity of the gas $v\rho$ is equal to the mass rate of decomposition of the backing, while on the lower surface of the facing it vanishes, i.e.,

when $y = \Delta$ $v\rho = (v\rho)_b$,

when $y = 0$ $v\rho = 0$.

Let us agree that the variation in $v\rho$ across the gap may be represented by

$$v\rho = (v\rho)_b \bar{y}^n, \quad \text{where} \quad \bar{y} = y/\Delta, \ n > 0. \tag{2}$$

From the results of isothermal solutions, it may be assumed that the value of n is close to unity.

Substituting (2) into (1) and integrating, we obtain

$$\frac{dt}{dy} = D\exp(-k\bar{y}^{n+1}), \quad \text{where} \quad k = (v\rho)_b c_p \Delta/\lambda(n+1). \tag{3}$$

The heat flux at the lower surface of the facing, $q_l = -\lambda \times \left(\frac{dt}{dy}\right)_{y=0}$ is equal to the external heat flux q_e.

$$D = -q_e/\lambda, \ q = -\lambda \frac{dt}{dy} = q_e \exp(-k\bar{y}^{n+1}). \tag{4}$$

At the surface of the backing $\bar{y} = 1$ heat is expended only on decomposition, and here

$$q_b = q_e \exp(-k) = (v\rho)_b F. \tag{5}$$

We recall, that, by the definition of k,

$$(v\rho)_b = k \frac{\lambda(n+1)}{c_p \Delta}. \tag{6}$$

Therefore, to find the rate of decomposition of the backing, we must solve the equation

$$q_e = \frac{F\lambda(n+1)}{c_p \Delta} k \exp k. \tag{7}$$

The external heat flux q_e depends on the still unknown temperature t_e of the external surface of the facing:

$$q_e = \alpha(t_g - t_e). \tag{8}$$

This is the heat flux received by the facing

$$q_e = \lambda_0 \frac{t_e - t_l}{\delta}. \tag{9}$$

Therefore

$$q_e = \alpha \frac{t_g - t_l}{1 + \mathrm{Bi}_0}, \quad \text{where} \quad \mathrm{Bi}_0 = \frac{\alpha\delta}{\lambda_0}. \tag{10}$$

The temperature of the lower surface of the facing t_l is found by integrating (4):

$$t_l - t_b = \frac{q_e \Delta}{\lambda} I, \quad \text{where} \quad I = \int_0^1 \exp(-k\bar{y}^{n+1})\, d\bar{y}. \tag{11}$$

By substituting $s = k\bar{y}^{n+1}$ we can express I in terms of an incomplete gamma function:

$$I = mk^{-m} \int_0^k \exp(-s)\, s^{m-1} ds = mk^{-m} \gamma(m, k), \tag{12}$$

where $m = 1/(n + 1)$.

The incomplete gamma function is related to the degenerate hypergeometric function:

$$\gamma(m, k) = m^{-1} k^m \exp(-k) M(1, 1+m, k). \tag{13}$$

It follows from (10)-(13) that:

$$\frac{t_l - t_b}{t_g - t_e} = \mathrm{Bi} \exp(-k) M(1, 1+m, k), \quad \text{where } \mathrm{Bi} = \alpha\Delta/\lambda,$$

$$q_e = \frac{\alpha(t_g - t_b)}{1 + \mathrm{Bi}_0 + \mathrm{Bi}\exp(-k) M(1, 1+m, k)}. \tag{14}$$

Substituting (14) into (7), we obtain an equation for k, and hence the desired mass decomposition rate of the backing $(v\rho)_b$,

$$\frac{1 + \mathrm{Bi}_0}{\mathrm{Bi}} \exp k + M(1, 1+m, k) = \frac{m}{k} \frac{c_p(t_g - t_b)}{F}. \tag{15}$$

Special cases. 1. The permissible facing temperature t_* is less than that of the surrounding medium t_g.

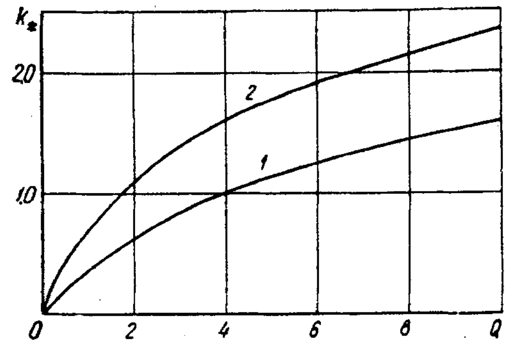
To obtain a real solution, we must make sure that the outer surface of the facing does not reach a dangerous value. With the condition $t_e \leq t_*$, (8), (7) and (15) take the form:

$$q_e \geqslant q_* = \alpha (t_g - t_*), \tag{8'}$$

$$\mathrm{Bi} = \frac{\alpha\Delta}{\lambda} \leqslant \frac{F}{c_p (t_g - t_*)} \frac{k}{m} \exp k, \tag{7'}$$

$$M(1.1 + m, k) \leqslant \frac{m}{k} Q, \tag{15'}$$

where $Q = \frac{c_p}{F} [t_* - t_p - (t_g - t_*) \mathrm{Bi}_0]$.



k_* as a function of Q for probable values of m: 1 – m = 0.5 (n = 1), 2 – m = 1 (n = 0)

The maximum permissible values of k, found by solving the equation

$$M(1.1 + m, k) = mQ/k, \tag{15''}$$

have been denoted by k_* and are shown in the figure*. From (7) the maximum permissible gap is

$$\Delta_* = \frac{F\lambda}{c_p q_*} \frac{k_*}{m} \exp k_* . \tag{7''}$$

From (5) is the minimum mass decomposition rate

$$(v\rho)_n \doteq \frac{q_*}{F} \exp(-k_*). \tag{5'}$$

2. The permissible facing temperature exceeds that of the surrounding medium t_g.

In this case it is permissible to heat the facing right up to the temperature t_g, and the gap may be arbitrary. When $t_e = t_g$, the heat flux q_e is determined by (9), (11)-(13), and (15) has the form

$$\frac{\lambda}{\lambda_0} \frac{\delta}{\Delta} \exp k + M(1{,}1 + m, k) = \frac{c_p (t_g - t_b)}{F} \frac{m}{k} . \tag{15'''}$$

The thermal conductivity of the gases λ is ten times less than that of the facing λ_0, and for actual coating parameters the solution of (15''') coincides with that of (15'') given in the figure. To avoid difficulties in determining the heat flux, the rate of decomposition may conveniently be found from (6):

$$(v\rho)_b = \frac{k}{m} \frac{\lambda}{c_p \Delta} . \tag{6'}$$

It is clearly inadvisable to keep the gap Δ constant. Values of v and Δ are determined from (6') by substituting $v = d\Delta/d\tau$. As a result we obtain

$$\frac{1}{v_b^2} - \frac{1}{v_1^2} = \frac{4\tau}{\beta^2} , \tag{16}$$

$$\Delta - \Delta_1 = \frac{\tau}{v_1} + \beta\tau^{0.5} , \tag{17}$$

where $v_1 = \frac{k}{m} \frac{\lambda}{c_p \Delta_1}$ is the initial rate of decomposition of the backing (for the initial gap), and $\beta = (2\lambda/c_p\rho_b) \cdot (k/m)^{0.5}$.

Provided the backing decomposition products will not enter into an exothermic reaction with the surrounding me-

*Calculated by A. A. Krasnyukii.

dium, it is convenient to release them through an aperture in the facing, or to use a porous facing for this purpose. In this case the decomposition products move normal to the surface, and $v\rho = (v\rho)_b$ = const.

For the usual condition $\frac{\lambda}{\lambda_0}\frac{\delta}{\Delta} \ll 1$ the solution for the case with a permeable facing coincides with the previous solutions, if we substitute m = 1 (n = 0). Other conditions being equal, the case m = 1 corresponds to the largest values of k and the least values of k/m. This means that the rate of decomposition under a permeable facing is less than under a continuous one, when n > 0 and m > 1 (5'), (6'), (16).

The efficiency of a coating with facing curtain may be judged from the following data.

When heated in air at t = 3000°K, $p = 30 \cdot 9.8 \cdot 10^4\ N/m^2$ and $\alpha = 80\ W/m^2 \cdot deg$, the mass erosion rate of quartz glass is 0.067 $kg/m^2 \cdot sec$, that of polyethylene 0.063, and that of polyethylene with a quartz facing 0.059.

In an inert gas with t = 3000°K, $p = 30 \cdot 9.8 \cdot 10^4\ N/m^2$ and $\alpha = 800\ W/m^2 \cdot deg$, after 100 sec of heating the distance of the 500°K isotherm from the starting surface of the coating will be 45 mm for graphite, 52 + 1 = 53 for polyethylene, and 7 + 8 = 15 mm for polyethylene with a graphite facing. In the last two cases the components indicate the depth of erosion and the thickness of the polyethylene layer heated above 500°K.

Under conditions of the first type, the decomposition products would burn on contact with the external flow. An impermeable facing is therefore used (n = 1, m = 0.5). The gap is kept constant (2 mm), and in operation the surface temperature of the facing is established at 1200°K, a temperature at which quartz glass does not erode. The backing and single layer coatings have approximately the same erosion. The main advantage of the coating with a facing curtain in this case is the fact that the contour remains unchanged.

Under conditions of the second type, the use of a porous facing (n = 0, m = 1) is favorable. A graphite facing may be safely heated right up to the temperature of the external stream; therefore the gap is not controlled. The initial value of the gap is $\Delta_1 = 0$. In 100 sec the backing is eroded by 7 mm, while for single-layer polyethylene the erosion is 52 mm. The graphite is heated without erosion. In spite of there being a noticeably heated layer, the composite coating is more effective than the single-layer coatings. If the heat transfer coefficient is greater or the heating time is less, the backing is not heated to the same depth, and the advantage of the coating with a facing curtain is considerable.

For a large heat flux it is difficult to achieve a gap $\Delta < \Delta_*$ to guard against overheating of the facing, because the values of Δ_* are too small. For example, for polyethylene with a quartz facing under conditions of the second type, $\Delta_* = 0.2$ mm. In such cases the permissible value of the gap may be increased at the expense of the thermal conductivity to values acceptable from the construction point of view.

A layer of porous material is inserted between the facing and the backing, and the backing is placed close to this intermediate layer. A porous gap is formed with higher thermal conductivity than that of the pure gas. For example, graphite with porosity 0.75-0.5 increases the thermal conductivity of the gap 20-50 times. According to (7"), the permissible value of the gap Δ_* increases in the same proportion. In this case the facing must, of course, be permeable. Use of a backing consisting of charring materials (principally phenol-formaldehyde, aniline-formaldehyde, and silicone resins) gives a similar effect. Then a porous gap is obtained due to incomplete gasification of the backing.

If there is no danger of overheating, and the facing may reach the temperature of the surrounding medium, the use of charring backings is unfavorable. Instead one should try to reduce the thermal conductivity of the gap, in order to achieve minimum decomposition of the backing (6'), (16), (17). In this respect a backing that gives complete gasification is preferable (polyethylene, polystyrene, teflon).

NOTATION

y – coordinate reckoned along normal from lower surface of facing; v – linear velocity in y direction; p – pressure; ρ – density; $v\rho$ – mass velocity in y direction; n – exponent in the parabolic law of variation of $v\rho$; m = 1/(n + 1); t – temperature; τ – time; c_p – specific heat at constant pressure; λ – thermal conductivity; F – heat of decomposition of backing; q – heat flux; α – heat transfer coefficient; δ – facing thickness; Δ – gap width; k – a root of Eq. (15). Subscripts: g – ambient gas; b – backing; e – upper surface of facing; l – lower surface of facing; parameters without subscripts refer to decomposition products.

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